Math 45 6.4 Special Products Day 1

Objectives:

Day 1
1) Factor difference of squares pattern
2) Factor perfect square trinomial pattern.
3) Recognize that the sum of squares is prime

Day 2
4) Factor difference of cubes
5) Factor sum of cubes

Chapter 5 Review.

Multiply

1. $(2x-5)^2$

$= (2x-5)(2x-5)$

$= 4x^2 - 10x - 10x + 25$

$= \boxed{4x^2 - 20x + 25}$

2. $(2x+5)^2$

$= (2x+5)(2x+5)$

$= 4x^2 + 10x + 10x + 25$

$= \boxed{4x^2 + 20x + 25}$

3. $(2x-5)(2x+5)$

$= 4x^2 + 10x - 10x - 25$

$= \boxed{4x^2 - 25}$

Chapter 6 Goal: Start with the end results and factor. Do the chapter 5 problem backward.

To do this, we need to get

from 4 to 2 $\quad x^2 \rightarrow x$

from 25 to 5.

and many others — square roots!

List perfect squares

1	$1^2 = 1$
2	$2^2 = 4$
3	$3^2 = 9$
4	$4^2 = 16$
5	$5^2 = 25$
6	$6^2 = 36$
7	$7^2 = 49$
8	$8^2 = 64$
9	$9^2 = 81$
10	$10^2 = 100$
11	$11^2 = 121$
12	$12^2 = 144$
13	$13^2 = 169$
14	$14^2 = 196$
15	$15^2 = 225$

* Memorize this list *

List Perfect Square Roots

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

$\sqrt{121} = 11$

$\sqrt{144} = 12$

$\sqrt{169} = 13$

$\sqrt{196} = 14$

$\sqrt{225} = 15$

For variables:

x	x^2
x^2	$(x^2)^2 = x^4$
x^3	$(x^3)^2 = x^6$
x^4	$(x^4)^2 = x^8$
x^5	$(x^5)^2 = x^{10}$
x^6	$(x^6)^2 = x^{12}$

If x is positive:

$\sqrt{x^2} = x$

$\sqrt{x^4} = x^2$

$\sqrt{x^6} = x^3$

$\sqrt{x^8} = x^4$

$\sqrt{x^{10}} = x^5$

$\sqrt{x^{12}} = x^6$

To take the square root Divide exponent by 2.

To take square root of a fraction, take square roots of numerator + denominator

$$\sqrt{\frac{x^4}{169}} = \frac{\sqrt{x^4}}{\sqrt{169}} = \frac{x^2}{13}$$

If exponent is not divisible by 2, the term is not a perfect square.

Factor completely.

(4) $4x^2 - 20x + 25$

<u>step 0:</u> Recognize <u>Perfect Square Trinomial</u>

- 3 terms
- last term is added
- first and last terms are perfect squares

<u>step 1:</u> Take square root of first term to find a

$a = \sqrt{4x^2}$

$a = \sqrt{4} \cdot \sqrt{x^2}$

$a = 2x$.

<u>step 2:</u> Take square root of last term to find b.

$b = \sqrt{25}$

$b = 5$

<u>step 3:</u> Substitute a and b into the

$$\boxed{\begin{array}{l}\text{Perfect Square Trinomial Formula} \\ a^2 - 2ab + b^2 = (a-b)^2 \text{ or } (a-b)(a-b)\end{array}}$$

$$= \boxed{(2x-5)^2} \text{ or } \boxed{(2x-5)(2x-5)}$$

<u>step 4:</u> Check the middle term by FOIL (or partial FOIL)

$(2x-5)(2x-5)$

$4x^2 - 10x - 10x + 25$

$4x^2 - 20x + 25$

<u>Note:</u> If the middle term does not match the original question, the trinomial cannot be factored.

(5) $4x^2 - 10x + 25$

gives $(2x-5)^2$ also!

but the middle term of $(2x-5)^2$ is $-20x$, not $-10x$, so it does not match.

$4x^2 - 10x + 25$ is $\boxed{\text{PRIME}}$.

⑥ $4x^2 + 20x + 25$

step 0: Recognize Perfect Square Trinomial

- 3 terms
- last term is added
- first and last terms are perfect squares.

> NOTE: The rules for signs that we learned in 6.2 also apply to these.
> product 25 is positive ⇒ factors have same sign.
> middle term 20x is positive ⇒ both positive.

steps 1 & 2 same as ④ and ⑤: $a = 2x$, $b = 5$

step 3: Substitute a and b into the

> Perfect Square Trinomial Formula
> $a^2 + 2ab + b^2 = (a+b)^2$ or $(a+b)(a+b)$

$= \boxed{(2x+5)^2}$ or $\boxed{(2x+5)(2x+5)}$

step 4: check middle term.

$(2x+5)(2x+5)$

$10x + 10x = 20x$ ✓

⑦ $4x^2 - 25$

step 0: Recognize Difference of Two Squares

- 2 terms
- subtracted
- both are perfect squares

steps 1 & 2: same as ④ and ⑤ $a = 2x$, $b = 5$

step 3: Substitute a and b into the

> Difference of Two Squares Formula
> $a^2 - b^2 = (a-b)(a+b)$ or $(a+b)(a-b)$

$= \boxed{(2x-5)(2x+5)}$ or $\boxed{(2x+5)(2x-5)}$

(8) $4x^2 + 25$

step 0: Recognize Sum of Two Squares
- 2 terms
- added
- both are perfect squares

step 1: Write $\boxed{\text{PRIME}}$.

(9) $x^4 - 16$

step 0: difference of two squares.

step 1: $a = \sqrt{x^4} = x^2$ $\quad$ $a = x^2$

step 2: $b = \sqrt{16} = 4$ $\quad$ $b = 4$

step 3: $(x^2-4)(x^2+4)$

Factor completely!

step 4: recognize that $x^2 - 4$ is also a diff of sq.

$a = \sqrt{x^2} = x$

$b = \sqrt{4} = 2.$

$x^2 - 4 = (x-2)(x+2).$

step 5: recognize that x^2+4 is a sum of sq. and cannot be factored.

step 6: Write all factors:

$= \boxed{(x-2)(x+2)(x^2+4)}$

(10) $81x^2 + 49y^2$ $\quad$ sum of squares

$\boxed{\text{PRIME}}$

Want more detail? see next page.

(11) $x^4 - 1$

$=(x^2-1)(x^2+1)$ $\quad$ Same steps as #9.

$= \boxed{(x-1)(x+1)(x^2+1)}$

6.4-19 **Factor completely. If the polynomial is prime, state so.**

$81x^2+49y^2$

A. $(9x+7y)(9x-7y)$

B. $(9x+7y)^2$

C. Prime ✔

D. $(9x-7y)^2$

This has 2 terms and 2 squares, but it's a <u>sum</u>, not a difference.

Here are the only possibilities:

I $(9x+7y)(9x+7y)$ → both (+)

FOIL to check:

$81x^2+63xy+63xy+49y^2$

$=81x^2+126xy+49y^2$ ← middle term wrong!

II $(9x+7y)(9x-7y)$ → one (+), one (−)

FOIL to check:

$81x^2-63xy+63xy-49y^2$

$=81x^2-49y^2$ ← sign wrong!

III $(9x-7y)(9x-7y)$ → both (−).

FOIL to check:

$81x^2-63xy-63xy+49y^2$

$=81x^2-126xy+49y^2$ ← middle term wrong!

No other options exist that give perfect squares.

So a **SUM OF SQUARES is ALWAYS prime**

12 $ab^4 - 144a^3b^2$

GCF first!

$= ab^2\left(\frac{ab^4}{ab^2} - \frac{144a^3b^2}{ab^2}\right)$

$= ab^2(b^2 - 144a^2)$

Difference of Squares ⇒ But can't use a and b from the formula! Write the formula with different letters: $(x^2 - y^2) = (x-y)(x+y)$.

$x = \sqrt{b^2} = b$ $\qquad \underline{x = b}$

$y = \sqrt{144a^2} = \sqrt{144} \cdot \sqrt{a^2} = 12a$ $\qquad \underline{y = 12a}$

$(b - 12a)(b + 12a)$

Write all factors

$= \boxed{ab^2(b-12a)(b+12a)}$

13 $1 - s^8$

Difference of squares #1

$a = \sqrt{1} = 1$ $\qquad a = 1$

$b = \sqrt{s^8} = s^4$ $\qquad b = s^4$

$= (1 - s^4)(1 + s^4)$

$1 + s^4$ = sum of squares (prime)

$1 - s^4$ = difference of squares #2

$a = \sqrt{1} = 1$ $\qquad a = 1$

$b = \sqrt{s^4} = s^2$ $\qquad b = s^2$

$= (1 - s^2)(1 + s^2)(1 + s^4)$

$1 + s^2$ = sum of squares (prime)

$1 - s^2$ = difference of squares #3

$a = \sqrt{1} = 1$ $\qquad a = 1$

$b = \sqrt{s^2} = s$ $\qquad b = s$

$= \boxed{(1-s)(1+s)(1+s^2)(1+s^4)}$

6.4.67 **Factor the polynomial completely.**

(14) $z^6w^2 - z^4w^4$

Select the correct choice below and fill in any answer boxes within your choice.

○A. $z^6w^2 - z^4w^4 =$

○B. The polynomial is prime.

GCF first!

smallest exp for z $\rightarrow z^4$

smallest exp for w $\rightarrow w^2$

$GCF = z^4w^2$

Factor out GCF

$= z^4w^2\left(\frac{z^6w^2}{z^4w^2} - \frac{z^4w^4}{z^4w^2}\right)$

$= z^4w^2(z^2 - w^2)$

Check factors to see if they can be factored —

$(z^2 - w^2)$ is a difference of squares $\begin{cases}\sqrt{z^2} = z \\ \sqrt{w^2} = w\end{cases}$

$= \boxed{z^4w^2(z-w)(z+w)}$

M45 6.4 Day 1

(15) $36x^2 - 84x + 49$

$= \boxed{(6x - 7)(6x - 7)}$

Notice perfect squares

$\sqrt{36x^2} = 6x$

$\sqrt{49} = 7$

Signs (−)(−)

check by FOIL!

$= 36x^2 - 42x - 42x + 49$

$= 36x^2 - 84x + 49$ ✓

(16) $64x^2 + 48x + 9$

$\sqrt{64x^2} = 8x$

$\sqrt{9} = 3$

$= \boxed{(8x + 3)(8x + 3)}$

$= 64x^2 + 24x + 24x + 9$ check by FOIL

$= 64x^2 + 48x + 9$ ✓